Department of Mathematics and Scientific Computing

National Institute of Technology Hamirpur

MA 812: Numerical and Statistical Methods

Time: 2:00hrs   Marks 50

Attempt any five questions. Marks are uniformly distributed.

1. Solve $10x + 2y + z = 9$, $-2x + 3y + 10z = 22$ $x + 10y - z = -22$ using Jacobi and Gauss Seidal Method upto three iterations. Also write iteration equations for both methods.

    b)   Solve   the   system   of   equations $x + y + z + w = 2, x + y + 3z - 2w = -6,$ $2x + 3y - z + 2w = 7$, $x + 2y + z - w = -2$ using elimination method.

2. A) Discuss iteration method to solve non-linear equations. Also state and prove convergence criteria of iteration method to solve non-linear equation.

    b) Find root of $x^x = 100$ correct to two decimal places using Newton Raphson method.

3. a) Function $u(x, y)$ satisfies $\nabla^2 u = 0$ and boundary conditions $u(x,0) = 0, u(x,4) = 8 + 2x$,

    $u(0, y) = y^2/2, u(4, y) = y^2$. Express it(pde) in terms of system of linear equation using step unit size in each direction and find the initial values.

    b) Evaluate $\int_0^1 \frac{dx}{1+x}$ using Romberg's method (step size 0.5, 0.25, 0.125). Hence find the approx..

        value of $\log_2 x$.

4. A) Use 4th order Runge Kutta method to find $y(0.1)$ given that $\frac{dy}{dx} - x^2 - y = 4$, $y(0) = 1$. Also compute exact value and hence find error.

    b) Derive standard five point formula to solve partial differential equations. Hence write five point diagonal formula. Explain the application of iteration (Jacobi and Gauss Seidal) process to find improved solution

5. a) Find Normal equations to fir a quadratic curve to given data. Hence deduce Normal equations for linear curve. Also discuss the process to fit i) power curve - $y = Ax^n$, ii) exponential curve $y = AB^x$ iii) y=ax + b/x using linear curve fitting.

b) Find $\dfrac{dy}{dx}$ & $\left(\dfrac{dy}{dx}\right)_{x=x_0}$ using Newton's forward interpolation formula. Using it find initial acceleration and acceleration at t = 2s using entire data given below for a moving object.

Time (sec)	0	5	10	15	20
Velocity (m/s)	0	3	14	69	228

6. Define probability function, Probability density function, cumulative function and its properties.

Hence find value of 'a' if $f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ 3a - x, & 2 \le x \le 3 \\ 0, & elsewhere \end{cases}$ is probability density function.

Hence find its cdf.

b) Define error, relative error, percentage error. Derive formula to compute error in volume of cylinder with radius 'r' and height 'h'.

c) State Lagrange's interpolation formula. Hence simplify it when data is equi-spaced. Using Lagrange's interpolation formula resolve $\dfrac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ into partial fractions.