Roll No.:................

National Institute of Technology, Hamirpur (HP)

End Semester Examination (16th Dec.-2020)

[Class: M.Sc.-Mathematics and Computing (1st Semester)]

Title of the Course: Differential Equations

Course Code: MA-613

Time: 02.00 Hours ***Maximum Marks: 50***

Note : Attempt all questions.

Topic: Existence and Uniqueness Theory

Q.No.1.: Calculate the first three approximations to the solutions of **(6)**

$$\frac{dy}{dx} = e^x + y^2, \quad y(0) = 0$$

by a method of successive approximation

Topic: Series Solution

Q.No.2.: To show that $P_n(x)$ is coefficient of h^n in the expansion in ascending powers of $(1-2hx+h^2)^{-1/2}$. **(3+3)**

Also express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$

in terms of Legendre's polynomials.

Q.No.3.: **(3+3)**

To prove that (i) $\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{(1-x^2)}}\,dx = \begin{cases} 0 & m \neq n \\ \pi/2 & m = n \neq 0 \\ \pi & m = n = 0 \end{cases}$

and (ii) $\int_{-1}^{1} \frac{U_m(x)U_n(x)}{\sqrt{(1-x^2)}}\,dx = \begin{cases} 0 & m \neq n \\ \pi/2 & m = n \neq 0 \\ \pi & m = n = 0 \end{cases}$

Topic: Sturm-Liouville Boundary Value Problems

Q.No.4.: Discuss the expansion of a function in a series of Orthonormal Functions. **(3+3)**

Obtain the formal expansion of the function 'f' , where f(x) = x, $0 \le x \le \pi$ in the series of orthonormal characteristic function $\{\phi_n\}$ of the Sturm-Liouville problem:

$$\frac{d^2y}{dx^2} + \lambda y = 0,$$
$$y(0) = 0, y(\pi) = 0$$

Topic: Non-Linear Differential Equations

Q.No.5.: Describe the different kinds of critical points. **(8)**

Q.No.6.: Determine the nature of critical point of the system **(3+3)**

$$\frac{dx}{dt} = x + y,$$
$$\frac{dy}{dt} = 3x - y.$$

Also find the general solution of the differential equation governing the path.

Topic: Partial Differential Equations

Q.No.7.: Discuss Charpit's method. **(3+3)**

And hence solve the following non-linear partial differential equations (Charpit's method):

$$px + qy = pq\,.$$

Q.No.8.: Derive Monge's (subsidiary) equations. **(3+3)**

And hence solve the following non-linear partial differential equation of second order (Monge's method):

$$(q + 1)s = (p + 1)t\,.$$

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