Department of Mathematics and Scientific Computing

National Institute of Technology Hamirpur

MA 612: Abstract Algebra(End Sem Exam-2020)

Time: 2:00 h   Marks 50

Attempt all questions

1. Prove that set of first m-non-negtive integers is a abelian group for composition being addition modulo m i.e. $a +_m b = r, 0 \leq r < m$, where r is least non negtive remainder when a+b is divided by m.
2. The necessary and sufficient conditions for a non empty subset K of a field F to be subfield of F are $a, b \in K \Rightarrow a + b \in K, -a \in K \ and \ ab^{-1} \in K \ if \ b \neq 0$.
3. Intersection of two subgroups is a subgroup but union of two subgroups is a subgroup iff one is contained in other.
4. Let N be a normal subgroup of group G and H be subgroup of G. Prove that i) $H \cap N$ is normal Subgroup of H and ii) HN is subgroup of G.
5. Let M be a finite extension field of a field K and K be a extension field of a field F then M is extension field of F and $[M : F] = [M : K][K : F]$
6. Let F be any subfield of K and G be any group of automorphism of K then Fixed field of G is a subfield of K. or

   If N and H are Normal subgroups of a group G then HN is also normal subgroup of G
7. If H and K are subgroups of group G then HK is subgroup of group G iff HK=KH
8. The necessary and sufficient condition for non empty subset S of a ring R to be subring of R are i) $a - b \in S, \ ii) \, ab \in S \ for \ all \ a, b \in S$
9. Define ideal of ring Hence show that intersection of any arbitrary family of ideals of a ring is itself an ideal.
10. Let G be a finite group and p be a prime number. Let that $p^m \mid o(G) \ and \ p^{m+1}$ don't divides o(G) Then there will be a subgroup of G of order p^m

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