National Institute of Technology Hamirpur
Department of Mathematics & Scientific Computing
MA 203: Engineering Mathematics III
End-Term Examination
Academic Year: 2020-2021

Time: 2:00 hrs **Max. Marks: 50**

Instructions:

- *All questions are compulsory. The marks allotted to a question are indicated against it.*
- *Your single **pdf file** to be uploaded be named as **RollNumberMA203**.*
- *First page of the answer sheet shall have Full Name, Roll Number, Subject Name, Subject Code, Programme, Semester, Name of the Department, Date of Exam and No. of Pages. Second page and onwards shall have Roll Number on the top and signature at the bottom along with **page number**. Pages without signature or pages with tempered signature will not be evaluated.*
- *You will be given 15 Minutes extra for scanning and sending the pdf file of answer sheet. Submission after the due time will attract penalty.*
- *Make sure that pages are visible and in correct order and orientation.*

---

1. Write the most appropriate answer: [5×2=10]

(i) Consider the Runge-Kutta method of the form

$$y_{n+1} = y_n + ak_1 + bk_1$$

where $k_1 = hf(x_n, y_n)$ and $k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$ to approximate the solution of the initial value problem

$$y'(x) = f(x, y(x)),\ y(x_0) = y_0.$$

Which of the following choice of a, b, α and β yields a second order method?

- $a = 1/2, b = 1/2, \alpha = 1, \beta = 1$
- $a = 1/4, b = 3/4, \alpha = 2/3, \beta = 2/3$
- $a = 1, b = 1, \alpha = 1/2, \beta = 1/2$
- $a = 1, b = 1, \alpha = 1, \beta = 1$

(ii) With reference to the finite difference operators, choose the most appropriate option:

- $\delta = E^{-1/2}\Delta$
- Both are true
- $\mu + \frac{1}{2}\delta = E^{1/2}$
- Both are false

(iii) Which of the following is *NOT* correct:

- The convergence of Gauss-Seidel method is more rapid than the Gauss-Jacobi method
- The sufficient condition for convergence of the Gauss-Jacobi method is that the system of equations is diagonally dominant
- The necessary and sufficient condition for convergence is that the spectral radius of the iteration matrix H is less than one
- None of these

(iv) For the curve $C:\ |z| = 1$, which of the following is *not* correct:

- $\oint_c e^z dz = 0$
- $\oint_c \bar{z} dz = 0$
- $\oint_c \frac{1}{z-2} dz = 0$
- None of these

(v) Behold the following statements:

A: Annulus is a simply connected domain

B: Infinitely many terms in principal part of the Laurent series of a function gives rise to a pole

- Only A is true
- Both A and B are true
- Only B is true
- None of these

2. (i) Perform two iterations of the Gauss-Jacobi iteration method for solving the system of equations:

$$\begin{pmatrix} 1 & 4 & 3 \\ 6 & 1 & 2 \\ 2 & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 8 \end{pmatrix}.$$

Take the initial approximation as $x^{(0)} = (1.3, -1.9, 0.8)^T$. [4]

(ii) Find the sum of the following series by using $C + iS$ technique [4]

$$1 + x\cos\theta + \frac{x^2}{2!}\cos 2\theta + \frac{x^3}{3!}\cos 3\theta + ...$$

(iii) If $f(z)$ is analytic function of z, show that [4]

$$\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2.$$

3. Find the correct value of $(30)^{-1/5}$, accurate upto four decimal places by Newton-Raphson method. For initial approximation, use the first iteration of bisection method in the interval $(0.1, 1)$. [7]

4. Find the all possible Taylor and Laurent series expansions of the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ about the point $z = 1$ in the region $2 < |z-1| < 3$. [7]

5. Using fourth order Adam predictor-corrector method, find $y(1.4)$ for the initial value problem,

$$y' = x^2(1 + y^2),\ y(1) = 1,$$

taking $h = 0.1$. Use Euler's method to calculate starting values. [7]

6. Find the residues at all the singular points of $f(z) = \frac{1}{z^3+z^5}$. Also evaluate the integral of $f(z)$ in $C: \ |z - i| = 3/2$, using the residue theorem. [7]

****************************** The End ******************************