National Institute of Technology Hamirpur

Electrical Engineering Department

End Semester Theory Examination: *M Tech* (*Signal Processing and Control*) (Dec. 2020)

Course Name: Probabilistic Methods for Signal & System Analysis **Course Code:** EE-632

Time: 2 Hours **Maximum Marks:** 50

Instructions:

Each student is required to write his/her **Name, Roll No, Subject Name and Subject Code on top of first sheet** and put **Signature with Date at the bottom of each sheet** of the answer booklet.

After the examination time is over, the students may be given extra 10 minutes to scan and upload their answer booklets on Google Classroom or send back on subject teacher's Email Id. Further, delay in submission by a student may lead to deduction in marks or rejection of whole answer booklet.

Q. 1 (a) Random variables X and Y have the joint PMF $p_{X,Y}(x,y)=\begin{cases} cxy & x\in\{1,2,4\},\ \text{and } y\in\{1,3\} \\ 0 & otherwise \end{cases}$ where c is a constant. What is $P(Y < X)$?

(b) Suppose the waiting time until the next bus at a particular bus stop is exponentially distributed, with parameter $\lambda=\frac{1}{15}$. Suppose that a bus pulls out just as you arrive at the stop. Find the probability that you wait between 15 and 30 minutes for a bus.

(c) Let a continuous random variable X be uniformly distributed over the interval $[-1, 1]$. Derive the PDF $f_Y(y)$ where $Y = \sin(\pi X/2)$

(d) Suppose $M_X(s)=\frac{1}{3}\frac{1}{1-s}+\frac{2}{3}\frac{3}{3-s}$ is the Moment Generating Function of random variable X. Find the *pdf* of X

(e) Imagine that you first roll a fair 6-sided die and then you flip a fair coin the number of times shown by the die. Letting X denote the number of these flips that come up heads, find $E[X]$.

(f) Given the information $E[X] = 7$ and $var(X) = 9$, use the Chebyshev inequality to find a lower bound for $P(4\le X\le 10)$

(g) The record of weights of the male population follows the normal distribution. Its mean and standard deviations are 70 kg and 15 kg respectively. If a researcher considers the records of 50 males, then what would be the mean and standard deviation of the chosen sample? **(Hint:**Use the results of CLT**)**

(h) Let $X(t)$ be a white Gaussian noise with PSD $S_X(f)=\frac{N_0}{2}$. Assume that $X(t)$ is input to an LTI system with impulse response $h(t)=e^{-t}u(t)$. Let $Y(t)$ be the output, find its average power i.e. $E\left[Y^2(t)\right]$.

(i) Prove that $E\left[(X\pm Y)^2\right]=\sigma_X^2+\sigma_Y^2\pm 2\rho\sigma_X\sigma_Y+(\mu_X\pm\mu_Y)^2$

(j) Consider the sine wave process $X(t)=A\sin(\omega t+\theta)$, where Θ is a uniform random variable over $(0, 2\pi)$ and A, $\acute{\omega}$ are constants. Check whether the process $X(t)$ is wide-sense stationary or not. **(Hint:** check WSS conditions**)** **(10×3=30)**

Q.2 The autocorrelation function for a stationary process $X(t)$ is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean and variance of $X(t)$ and also the mean value of the random variable $Y = \int_{\tau=0}^{2} X(t)\,dt$ **(6)**

Q.3 State and prove Central Limit Theorem (CLT) **(6)**

Q.4 Consider the random process $X(t)$ defined by $X(t) = Y\cos(\omega t),\ \ t \geq 0$ where ω is a constant and Y is a uniform random variable over (0, 1).

Find: **(a)** The autocorrelation function $R_X(t, s)$ of $X(t)$

(b) The autocovariance function $C_X(t, s)$ of $X(t)$

(8)

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