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National Institute of Technology Hamirpur
Department of Mathematics & Scientific Computing
MA 211: Discrete Mathematics
End Semester Examination, November 2023

20/11/23 (M)

Time: 3:00 hrs **Max. Mark: 50**

Note: All questions are compulsory.

1. Define the following terms with examples: $[1 \times 5 = 5]$
   (a) Floor function
   (b) Big-Theta
   (c) Nested Quantifier
   (d) Full-adder
   (e) Isomorphic graphs

2. (a) Using a Karnaugh map, simplify each boolean expression: [2]
   $$E_1 = wxyz + wxyz' + wxy'z' + wxy'z + w'xyz + w'xy'z$$
   (b) How can this English sentence be translated into a logical expression ? [2]
   "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."
   (c) Use set builder notation and logical equivalences to establish the second De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$, by quoting every law used. [2]
   (d) Prove that the complete graph $\mathbf{K}_5$ is nonplanar. [2]
   (e) Find the DNF of the boolean function $f(x, y, z) = x + yz$, using the laws of boolean algebra, by quoting every law used. [2]

3. Let S be the set of all bit strings. Suppose that R_3 is the relation on S such that sR_3t either when $s = t$ or both s and t are bit strings of length 3 or more that begin with the same three bits. What are the sets in the partition of the set of all bit strings arising from the relation R_3 on the set of all bit strings ? [5]

4. Solve the LNHRRWCCs: $a_n = 5a_{n-1} - 6a_{n-2} + 8n^2$, where $a_0 = 4$ and $a_1 = 7$. [5]

5. Using the laws of logic, simplify the boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$, by quoting every law used. [5]

6. Prove that the number of leap years ℓ after 1600 and not exceeding a given year y is given by $\ell = \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor - 388$. [5]

7. Draw the Hasse diagram for the poset $(A, |)$, where $A = \{1, 2, 3, 6, 8, 12\}$ and $|$ denotes the divisibility relation. Find the minimal, maximal, least and greatest elements. [5]

8. Using generating functions, solve the Fibonacci recurrence relation $F_n = F_{n-1} + F_{n-2}$, where $F_1 = 1 = F_2$. [5]

9. (a) What is the chromatic number of the graph C_n, where $n \geq 3$? [2]
   (b) The given table lists the students taking the various courses at NIT Hamirpur. The examination cell would like to develop a conflict-free final exam schedule using as few time slots as possible. How can we help her ? [3]

Course A	Course B	Course C	Course D	Course E	Course F	Course G
Prakash	Parveen	Yatin	Prakash	Prakash	Jyoti	Jyoti
Ankit	Vandana	Vandana	Om	Parveen	Om	Prakash
Parveen	Kanika	Anjali	Sushma	Sanjeev	Anjali	Sanjeev
Om	Sahil	Kanika	Saini	Sharma	Kumar	Toshith
Sushma	Upasana	Kumar	Sahil	Kanika	Upasana	Saini
Kapil						

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