Dr. Rakesh Sharma 22/11/2023 (81) (B)

National Institute of Technology, Hamirpur (H.P.)

Roll No :

End Semester Examination : B.Tech/ Dual Degree (Dec 2023)

EC-313 : Digital Signal Processing

Branch : E&CE

Time : 3 Hrs

Semester: 5^{th}

Max. Marks : 50

Note:

All symbols used have their usual meanings.

Assume necessary data, if any.

Attempt all parts of the question at one place only.

1.

[10 Marks]

(a) Determine whether or not the following signals are periodic:

$$x_1(n) = 2\exp(j(n/6 - \pi)) + \exp(j6\pi n)$$

and

$$x_2(n) = \cos(3\pi n/6)\cos(\pi n/8).$$

If periodic find the fundamental period.

(b) Compute the convolution $y(n) = x(n) * h(n)$ for the following pair of signals.

$$x(n) = \begin{cases} 1 & \text{for } n = -2, 0, 1 \\ 2 & \text{for } n = -1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$$

2.

[10 Marks]

(a) Determine the z-transform of the signal $x(n) = (\frac{1}{2})^n u(n+2) + n3^n u(-n-1)$.

(b) Consider the following LTI system

$$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2).$$

Determine:

i. The impulse response.

ii. The zero state step response.

3.

[10 Marks]

(a) A signal x(n) has the discrete time Fourier transform $X(\omega)$:

$$X(\omega) = \frac{1}{(1 - ae^{-j\omega})}.$$

Determine the Fourier transforms of the following signals using properties of DTFT.

i. $x(2n+1)$

ii. $e^{\frac{\pi n}{2}} x(n+2)$

iii. $x(n) * x(-n)$

iv. $x(-2n)$

(b) A 8-point sequence $x(n)$ has the Fourier series coefficients:

$$C_k = \begin{cases} \sin(\frac{k\pi}{3}), & \text{for } 0 \leq k \leq 6 \\ 0, & \text{for } k = 7. \end{cases}$$

Calculate the energy of $x(n)$.

4. [10 Marks]

(a) The computational time for calculating N-DFT is determined by the computational time required to evaluate number of complex multiplications and complex additions. If a complex multiplication requires 2 μs and a complex addition requires 1 μs. Then, evaluate the computational time required to calculate 2048-point DFT using direct computation and radix-2 decimation in time or frequency FFT algorithm.

(b) Compute the 8-point DFT using radix-2 decimation in time FFT algorithm of the following signal:

$$x(n) = \cos(\pi n) \quad 0 \leq n \leq 7.$$

5. [10 Marks]

(a) Obtain direct form I, Direct form II, cascade, and parallel structures for the following system:

$$y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2).$$

(b) For the sequences

$$x_1(n) = \cos\frac{\pi}{4}n, \quad x_2(n) = \sin\frac{\pi}{4}n, \quad 0 \leq n \leq 7.$$

Calculate a 8-point circular convolution of $x_1(n)$, and $x_2(n)$.

Best of Luck