Dr Vimal Sharma 8/5/2023

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162

NATIONAL INSTITUTE OF TECHNOLOGY HAMIRPUR
END TERM EXAMINATION
B. Tech. 2nd year, 4th Sem.
Mathematical Physics (PH-223)

Time: 3 Hrs.

MM: 50

Note: *Students are advised to be careful while attempting the question paper and do not leave any question unanswered. Marks for each question are mentioned in front of the question.*

Section A

Q 1 Show that the function f(z) = Sin(z) Cosh(y) + i Cos(x) Sinh(y) is continuous as well as analytic everywhere. (3)

Q 2 Find the analytic function whose real part is x^3-3xy^2. (3)

Q 3 Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center at 0. (5)

Q 4 Use Residue integration method to show that: $\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2\sqrt{2}}$ (5)

Section B

Q 5 Define: (i) Ordinary Point (ii) Singular Point and (iii) Regular Singular point of y" + P(x)y' + Q(x)y = 0. Obtain the series solution of the equation y" + x2y' +2xy = 0, about x = 0. (6)

Q 6 In a Maxwellian distribution the fraction of particles of mass m with speed between v and v + dv is

$$\frac{dN}{N} = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) v^2\, dv.$$

Where N is the total number of particles, k is Boltzmann's constant, and T is the absolute temperature. The average or expectation value of v^n is defined as. Show that

$$\langle v^n \rangle = \left(\frac{2kT}{m}\right)^{n/2} \frac{\Gamma\left(\frac{n+3}{2}\right)}{\Gamma\left(\frac{3}{2}\right)}.$$

Here the distribution was in kinetic energy E = $mv^2/2$, with dE = mvdv. (6)

Section C

Q 7 Use the basic recurrence relation, to prove the following formulas:

$$J_n(x) = J'_{n+1} + \frac{n+1}{x} J_{n+1}(x) \qquad J_{1/2}(x) = \left(\frac{2}{\pi x}\right)^{1/2} Sin(x)$$

(6)

Section D

Q 8 Use Laplace transform method to solve the initial value problem for a damped mass-spring system acted upon by a sinusoidal force for some time interval: y" + 2y' + 2y = r(t), r(t) = 10 sin 2t if 0 < t < π and 0 if t > π; y(0) = 1, y'(0) = -5. (5)

Q 9 Develop the irreducible 2 × 2 matrix representation of the group of rotations (including those that turn it over) that transform a square into itself. Give the group multiplication table. (5)

Q 10 Do the three matrices

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Form a group (under matrix multiplication)? Add a minimum number of matrices to this set to make it a group. Find these necessary additional matrices and write down the multiplication table and classes. Is this group isomorphic to (E, C_4, C_4^2, C_4^3) or to (E, C_4^2, m_x, m_y) or to both? (6)