2023

National Institute of Technology Hamirpur Department of Mathematics & Scientific Computing

MA-322: Advanced Differential Equations, End Term Examination, May 2023

	i mene, End ferm Exam	1111a11011, $111ay 2023$
Time: 3:00 hrs	Roll Number:	Max Marks 50
T /		Wax. Wark: 50

Instructions: All questions are compulsory. Each question carries 5 marks.

1. Solve the initial value problem

$$(y + \sqrt{x^2 + y^2})dx - xdy = 0, \ y(1) = 0.$$

- 2. A tank initially contains 50 gallons of fresh water. Brine, containing 2 pounds per gallon of salt, flows into the tank at the rate of 2 gallons per minute and the mixture kept uniform by stirring, runs out at the same rate. How long will it take for the quantity of salt in the tank to increase from 40 to 80 pounds?
- 3. Find the solution y(t) of the initial value problem:

$$y'' + 4y' - 2y = 0; y(0) = 1, y'(0) = 2.$$

- 4. Find a differential equation which possesses e^x , $e^{-x} \cos 2x$ as solutions.
- 5. Classify the singular point(s) in the following differential equation and solve it using the method of Frobenius

$$x^{2}y'' + 3xy' + (1+x)y = 0.$$

6. Form the partial differential equation by eliminating the arbitrary function from

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$$

7. Find the integral surface of the partial differential equation:

$$x(y^{2}+z)p - y(x^{2}+z)q = (x^{2}-y^{2})z$$

containing the line x + y = 0, z = 1.

8. Find the complete integral of the following partial differential equation using Charpit's method:

$$u^2 = xypq$$

- 9. Use the method of separation of variables to solve the partial differential equation $u_{xx} = u_y + 2u$ where u(0, y) = 0, $u_x(0, y) = 1 + e^{-3y}$.
- 10. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, 0 < x < 1, t > 0 with the initial condition $u(x, 0) = 3 \sin 2\pi x$ and the boundary conditions u(0, t) = 0, u(1, t) = 0.

* * * * * * **