Dr Sampa Kumar Mish 4/5/2023

National Institute of Technology Hamirpur, HP

Department of Mathematics and Scientific Computing
MA-221: Linear Algebra and Applications
End Semester (Even) Examinations (B. Tech, 2^{nd} Year)
Academic Session : 2022 - 2023

Time: 3 hrs | Roll Number- | **Max. Mark: 50**

Note: All questions are compulsory. The marks allotted to a question are indicated against it. Symbols have their usual meaning.

1. (a) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$. Show that there exists a matrix P such that $PA = LU$. Determine the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, where L is a lower triangular matrix and U is an upper triangular matrix in each case. [6]

   (b) Define a basis of a vector space V over the field $\mathbb{F}$. Show that $\{(1,2,1),(3,1,2),(1,1,1)\}$ form a basis of $\mathbb{R}^3$. Further, write $(3,5,2)$ as a linear combination of the members of the above basis. [6]

2. (a) State the Rank - Nullity Theorem for the linear transformations. Define a map $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x,y,z) = (x-y, y-z, z-x)$. Show that T is a linear map. Further, verify the Rank - Nullity Theorem for the above linear map. [6]

   (b) Let V be a vector space over the field F. Define the dual space , V^*, of V. Let $V = \mathbb{R}^3$. Let $B = \{(1,0,0),(1,1,0),(1,1,1)\}$ is a basis of $\mathbb{R}^3$. Find the dual base of B in V^*. [6]

3. (a) Let V ve a vector space over some field $\mathbb{F}$. Define the Diagonalizability of a linear operator $T : V \to V$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x,y) = (7x+2y, -4x+y)$ be a linear map. Find a basis B of $\mathbb{R}^2$ with the property that matrix of T with respect to basis B is a diagonal matrix. Further, find the eigenvalues an eigenvectors of T. [6]

   (b) State the Spectral Theorem. Let $A = \begin{bmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ be a matrix. Then find an orthogonal matrix P such that P^tAP is a diagonal matrix. Further, find a real symmetric matrix B, if possible, such that $B^2 = A$. [6]

4. (a) State the Singular Value Decomposition(SVD) of a Matrix A of order $m \times n$. Determine the singular value decomposition of $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$. Also, describe the application of SVD on image processing. [6]

   (b) Write a short note on the application of
   (i) Linear system of equations on Graphs and Networks.
   (ii) Markov Chain's on population migration from urban to suburban and vice-versa.
   (iii) Inner product spaces. [8]

********** All the best **********