National Institute of Technology Hamirpur

Department of Mathematics & Scientific Computing

MA-203: Engineering Mathematics III, End Term Examination, May 2023

Time: 3:00 hrs | Roll No.:..... | Max. Mark: 50

Instructions: All questions are compulsory. Each question carries 5 marks.

1. Perform three iterations of the Gauss-Jacobi iteration method to solve the following system of equations:

$$6x + y + 2z = 6$$
$$x + 4y + 3z = -4$$
$$2x + y + 8z = 8$$

taking the initial approximation as $x = 0,\ y = 0,\ z = 0$. Consider four decimals in the computation.

2. Obtain the approximate value of $\frac{1}{\sqrt{7}}$ accurate to three decimal places, using Newton-Raphson method. Use 0.5 as the initial approximation.

3. Given

θ (in degree):	5	10	15	20	25
$\tan\theta$:	0.0875	0.1763	0.2679	0.3640	0.4663

Using Stirling formula, estimate the value of $\tan 16^o$. Use four decimal place representation in the computation.

4. The pressure and volume of a gas are related by the equation $pv^\gamma = k$, γ and k being the constants. Using least square method, fit this equation to the following set of observations:

p :	0.5	1.0	1.5	2.0	2.5	3.0
v :	1.62	1.00	0.75	0.62	0.52	0.46

5. Evaluate the double integral

$$\int_1^5 \int_1^5 \frac{dxdy}{\sqrt{x^2+y^2}}$$

using the trapezoidal rule with two subintervals on each axis. Use four decimal place representation in the computation.

6. Using Runge-Kutta method of fourth order, solve for y at $x = 1.2$ from the following initial value problem

$$\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x},\ y(1) = 0.$$

Take $h = 0.2$ and use four decimal place representation in the computation.

7. Given

$$\frac{dy}{dx} = x^2(1+y)$$

and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$ and $y(1.3) = 1.979$. Evaluate $y(1.4)$ by using Adams predictor-corrector method of fourth order. Use three decimal place representation in the computation.

8. (a) Identify the region $|z - 2i| < |z + 2i|$.
   (b) Find the real and imaginary parts of $\log((1+i)\log i)$, where log represents principal logarithm.

9. Show that the function $f(z) = \sqrt{|xy|}$ is not differentiable at origin even though the Cauchy-Riemann equations are satisfied at origin.

10. Find the following integral using residue theorem

$$\oint_{C:|z|=1} \frac{\cot z}{(z-1)^2(2z-1)}.$$