## $\mathfrak{N a t i o n a l}$ Institute of Technology Hamirpur ( $\mathcal{H}(\mathcal{P})$ <br> End Semester Examination <br> [Class: B.Tech. (2 ${ }^{\text {nd }}$ Semester) <br> Title of the Course: Engineering Mathematics-II <br> Course Code: MA-121

Maximum Marks: 50

## Time: 3 Hours

Note : Attempt all questions.

## Ordinary Differential Equations

Q.No.1.: Solve the following differential equations
(a) $(1+x y) y d x+(1-x y) x d y=0$.
(b) $(p x+y)^{2}=p y^{2}$.

Course Outcomes: CO1, CO2, CO3, CO4.
Q.No.2.: Solve the following simultaneous differential equations:

$$
\frac{\mathrm{dx}}{\mathrm{dt}}+5 \mathrm{x}-2 \mathrm{y}=\mathrm{t}, \frac{\mathrm{dy}}{\mathrm{dt}}+2 \mathrm{x}+\mathrm{y}=0
$$

being given $\mathrm{x}=\mathrm{y}=0$, when $\mathrm{t}=0$.

Course Outcomes: CO1, CO2, CO3, CO4.

## Partial Differential Equations

Q.No.3.: Solve the following Lagrange's linear partial differential equation:

$$
\left(x^{2}-y^{2}-z^{2}\right) p+2 x y q=2 x z
$$

Course Outcomes: CO1, CO2, CO3, , CO4.
Q.No.4.: Solve the following non-homogeneous linear partial differential equations:

$$
\left(\mathrm{D}^{2}-\mathrm{DD}^{\prime}+\mathrm{D}^{\prime}-1\right) z=\cos (\mathrm{x}+2 \mathrm{y})
$$

Course Outcomes: CO1, CO2, CO3, CO4.

## Transforms Theory

(Laplace Transforms, Fourier Transforms and $\mathbb{Z}$-Transforms)
Q.No.5.: (a) Find the Laplace transforms of
(i) $t \cos a t$, (ii) $t^{3} e^{-3 t}$, (iii) $\mathrm{t}^{-\mathrm{t}} \sin 3 \mathrm{t}$.
(b) Solve, by the method of Laplace transforms, the differential equation

$$
\begin{equation*}
\left(D^{3}-3 D^{2}+3 D-1\right) y=t^{2} e^{t} \tag{5}
\end{equation*}
$$

given that $\mathrm{y}(0)=1, \mathrm{y}^{\prime}=(0)=0, \mathrm{y}^{\prime \prime}(0)=-2$.
Course Outcomes: CO1, CO2, CO3.
Q.No.6.: (a) If $\mathrm{F}(\mathrm{s})$ is the complex Fourier transform of $\mathrm{f}(\mathrm{x})$, then find $\mathrm{F}\{\mathrm{f}(\mathrm{ax})\}, \mathrm{a} \neq 0$.
(b) If $\mathrm{F}_{\mathrm{s}}(\mathrm{s})$ is Fourier sine transforms of $\mathrm{f}(\mathrm{x})$, then show that

$$
\mathrm{F}_{\mathrm{s}}\{\mathrm{f}(\mathrm{x}) \sin \mathrm{ax}\}=\frac{1}{2}\left[\mathrm{~F}_{\mathrm{c}}(\mathrm{~s}-\mathrm{a})-\mathrm{F}_{\mathrm{c}}(\mathrm{~s}+\mathrm{a})\right] .
$$

Course Outcomes: CO1, CO2, CO3, CO4.
Q.No.7.: Solve (by Fourier transform) the following partial differential equation $\frac{\partial \mathrm{u}}{\partial \mathrm{t}}=2 \frac{\partial^{2} \mathrm{u}}{\partial \mathrm{x}^{2}}$, if $\mathrm{u}(0, \mathrm{t})=0, \mathrm{u}(\mathrm{x}, 0)=\mathrm{e}^{-\mathrm{x}}(\mathrm{x}>0)$ and $\mathrm{u}(\mathrm{x}, \mathrm{t})$ is bounded, where $x>0, t>0$.

Course Outcomes: CO1, CO2, CO3.
Q.No.8.: (i) Find $\mathbb{Z}\{n\}$,
(ii) Prove that $\mathbb{Z}\left\{r^{n} \cos n \theta\right\}=\frac{z(z-r \cos \theta)}{z^{2}-2 z r \cos \theta+r^{2}}$,
(iii) Find $\mathbb{Z}\left\{\mathrm{e}^{-\mathrm{at}}\right\}$,
(iv) Prove that $\mathbb{Z}\left\{e^{-a t} \cos b t\right\}=\frac{z e^{a T}\left\{z^{a T}-\cos b T\right\}}{z^{2} e^{2 a T}-2 z^{a T} \cos b T+1}$.

Course Outcomes: CO1, CO2, CO3, CO4.
Probability and Statistics
Q.No.9.: In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2 . Out of 1000 such samples, how many would be expected to contain at least 3 defective parts?
Course Outcomes: CO1, CO3, CO4.
Q.No.10.: In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S. D. of 60 hours. Estimate the number of bulbs likely to burn for
(a) more than 2150 hours,
(b) less than 1960 hours and
(c) more than 1920 hours but less than 2160 hours.

Given: Area against $\mathrm{z}=1.33$ in the table $=0.4082$.
Area against $\mathrm{z}=1.83$ in the table $=0.4664$.
Area against $\mathrm{z}=2.00$ in the table $=0.4772$.
Course Outcomes: C01, CO3, CO4, C05.

