Dr. Pawan Kr Sharma (160) 7/12/2022

Department of Mathematics

National Institute of Technology Hamirpur HP INDIA

End Semester Examination (Dec. 2022)

MA 812: Numerical and Ststistical Methods

Time: 3 hours **Marks 50**

All questions are compulsory.

1. a) Find Newton Cotes quadrature formula. Hence obtain Trapezoidal method. Also find expression for error in trapezoidal rule and Romberg integration formula.

   b) **Find the interpolating polynomial for $\frac{dy}{dx}$ from the data given below using**

   Newton's forward interpolation formula.

x	4	6	8	10
y	1	3	8	16

2. a) Explain iteration method to solve nonlinear equations. Hence state and prove the sufficient condition for its convergence.

   b) Compute $\int_0^1 \frac{dx}{1+x}$, by taking h = 1/2, 1/4 1/8 using Trapezoidal rule then apply Romberg integration and hence find the approximate value of log 2.

3. a) Write Fourier series expansion of function for arbitrary period, hence for period 2π, even odd functions, convergence and Sufficient conditions for Fourier series expansion.
   b) Derive normal equations for least square linear curve fitting Hence extend theses to quadratic curve. Discuss the process to fit exponential, power curve and Gas equation using linear curve fitting.

4. a) Given $\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} = x^3$, $y(1) = 1$, $y'(1) = 1$. Obtain the value of y at x = 1.1 and 1.3 using Taylor series method of order 5.

   b) Explain Binomial distribution. Obtains its recurrence formula. Hence find variance of this distribution

5. a) Solve $\frac{d^4y}{dx^4} - 16y = x$, for $y(0.25), y(0.5), y(0.75)$
   Given $y(0) = 0, y''(0) = 0, y(1) = 0, y'(1) = 0$ using finite difference approach.

   b) Solve Poisson's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -4(x + y)$ over square mesh with sides x = 0, y= 0, x = 4 and y = 4 with u = 0 on boundary taking mesh of unit length.