Dr. Subir Kumar Jain 2/12/2022

78

M

National Institute of Technology Hamirpur
Department of Mathematics & Scientific Computing
M.Sc.(Mathematics & Computing), Semester-III
MA 632: Numerical Analysis
End-Term Examination, November-December 2022

Time: 3:00 hrs | Max. Mark: 50

Note: All questions are compulsory.

1. (i) For the given Maclaurin series of e^x, [4x2=8]

$$e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + ...$$

If we want to compute $e^{0.01}$ with an error less than 10^{-12} , atleast how many terms are needed?

(ii) Using the infinity-norm, calculate the condition number of the following matrix,

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(iii) Solve $(\frac{\Delta^2}{E})e^x \frac{Ee^x}{\Delta^2 e^x}$, where Δ and E represents forward difference operator and shift operator, respectively. And, the interval of differencing is h.

(iv) Determine the step size h that can be used in the tabulation of $f(x) = sinx$ in the interval $[1, 3]$ so that the linear interpolation will be correct to four decimal places after rounding.

2. Determine the rate of convergence of the secant method. And also find a real root, correct to three significant figures, of the equation $x^3 - 2x - 5 = 0$ in the interval $[2, 3]$ using secant method. [6]

3. Using the Jacobi method, find all the eigenvalues and the corresponding eigenvectors of the matrix [6]

$$\mathbf{A} = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

4. The integral [6]

$$I = \int_0^3 \frac{dx}{x^2 + 2x + 10}$$

is split into two parts as

$$I = I_1 + I_2 = \int_0^2 \frac{dx}{x^2 + 2x + 10} + \int_2^3 \frac{dx}{x^2 + 2x + 10}$$

Apply two point Gaussian integration formula to each of I_1 and I_2 and hence find the value of I correct to atleast three decimal places.

5. Values of x (in degrees) and $\sin x$ are given in the following [6]

x:	15	20	25	30	35	40
$\sin x$:	0.2588190	0.3220201	0.4226183	0.5	0.5735764	0.6427876

Determine the value of $\sin 38^o$, using appropriate Newton's method.

79

6. Given the following values of $f(x)$ and $f'(x)$ [6]

x:	-1	0	1
$f(x)$:	1	1	3
$f'(x)$:	-5	1	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

7. Solve the initial value problem [6]

$$u' = -2tu^2$$

with $u(0) = 1$ and $h = 0.2$ on the interval $[0, 0.4]$. Use the fourth order Runge-Kutta method. Consider seven digits for computation.

8. A boundary value problem (BVP) is defined by [6]

$$\frac{d^2y}{dx^2} - y = 0$$

where $y(0) = 0$ and $y(2) = 3.62686$. With $h = 0.5$, use the finite difference method to solve the BVP. Also compute the absolute error at each nodal point. (Note: Use second order accurate central difference scheme to approximate the derivatives.)

****