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Roll No.:...............

National Institute of Technology, Hamirpur (HP)

End Semester Examination(Nov/Dec-2022)

[Class: M.Sc.(Mathematics & Computing) (3rd Semester)]

Title of the Course: Functional Analysis **Course Code: MA-631**

Time: 03.00 Hours ***Maximum Marks: 50***

Note : Attempt all questions.

1. State the following 5

   i. Hahn Banach Theorem
   ii. Open mapping Theorem
   iii. Closed Graph Theorem
   iv. Uniform Bounded Principle
   v. Banach Contraction Principle

2. Define the following 5

   i. Normal Operator
   ii. Summability in Normed Linear Space
   iii. Isometric Isomorphism
   iv. Hilbert Space
   v. Orthogonal Compliment
   vi. Orthonormal Set
   vii. Complete Orthonormal Set
   viii. Adjoint of an operator
   ix. Self-Adjoint operator
   x. Continuous Linear Transformation

3. Let H be a Hilbert space and T be a positive operator on H then, $I + T$ is non-singular. 5

4. If P is a projection on a Hilbert space H with range M and null space N, then $M \perp N$ if and only if P is self-adjoint and in this case $N = M^{\perp}$ 5

5. Let T be an operator on a Hilbert space H. Then, there exists a unique operator on T^* on H such that $(Tx, y) = (x, T^*y)\ \forall\, x, y \in H$ 5

6. (i) If T is a normal operator on a Hilbert space H, then the eigenspaces of T are pairwise orthogonal. 5

   (ii) Let X be a normed space then

   (a) addition and scalar multiplication are jointly continuous in X and

   (b) If x_n converges to x and y_n converges to y then ax_n+by_n converges to ax+by, where a, b are constants.

7. Let $\{e_i\}$ be an orthonormal set in Hilbert space H and if x is an arbitrary vector in H, then 5

$$(x - \sum(x, e_i)e_i) \perp e_j \text{ for each } j$$

8. Let N be a normed linear space and suppose two norms $\|.\|_1$ and $\|.\|_2$ are defined on N. Then these norms are equivalent if and only if there exists positive real numbers m and M such that 5

$$m\|x\|_1 \leq \|x\|_2 \leq M\|x\|_1$$

9. Let N be a non-zero normed linear space and let $S = \{x \in N : \|x\| \leq 1\}$ be a linear subspace of N. Then N is a banach space if and only if S is complete. 5

10 A normed linear space N is a Banach space iff every absolutely summable series in N is summable. 5