Dr Sriench Kumar (61) Mustare 2/12/2022

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National Institute of Technology Hamirpur, HP

MA-312: Number Theory and Abstract Algebra

B.Tech 5^{th} Semester (Mathematics and Computing)

End Semester Examinations (Odd)

Academic Session : 2022 - 2023

Max. Mark: 50

Time: 3 hrs

Note: All questions are compulsory. The marks allotted to a question are indicated against it. Symbols have their usual meaning.

1. (i) State the Wilson theorem. Use Wilson theorem to obtain the remainder when $27! \times 30 + 2$ is divided by 29. [2]

   (ii) Prove or disprove that in a commutative ring with identity, every prime ideal is maximal. [2]

   (iii) (a) Find $\phi(2002)$, where ϕ is the Euler's phi function. [1]

   (b) Find the number of subgroups of $\mathbb{Z}_{3003}$. [1]

   (iv) Prove or disprove that subring of an Euclidean domain is an Euclidean Domain. [2]

   (v) Write a field of order 4. [2]

   (vi) Prove or disprove that $\mathbb{Z}_{31}$ is an Integral Domain. [2]

   (vii) Determine the group of units of $\mathbb{Z}[i]$. [2]

2. (a) Determine an integer solution of $51X \equiv 12 \pmod{87}$. [3]

   (b) State the Euler's theorem for congruences. Use Euler's theorem to determine the last two digits in the decimal representation of 3^{256}. [3]

   (c) Find a least positive integer solution of the system $X \equiv 2 \pmod 3$, $X \equiv 1 \pmod 5$, $X \equiv 3 \pmod 7$, by using Chinese remainder theorem. [4]

3. (a) Define the divisor function τ. Show that divisor function is multiplicative. [3]

   (b) Define a finite continued fraction. Write $\frac{19}{51}$ in the form of a continued fraction. [3]

4. (a) Let p be a prime. Define a Sylow p-subgroup of a finite group G. Determine all Sylow 2-subgroups of S_4. [4]

   (b) Write all the normal subgroups of S_3. [2]

5. (a) Define a Principal Ideal Domain (PID). Show that any field F is a PID. [3]

   (b) Define a prime element in an integral domain. Prove or disprove that 2 is a prime in $\mathbb{Z}[\sqrt{-5}]$. [4]

6. (a) Define an splitting field of a polynomial over a field F. Determine the splitting field of $x^3 - 2$. [3]

   (b) Define a Galois extension of a field F. Show that $\mathbb{Q}(\sqrt{2})$ is the Galois extension of $\mathbb{Q}$. [4]

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