

**Instructions:** 

- All Questions are compulsory.
- Marks are given against each question.

**Q1.** The mean and variance of binomial distribution are 4 and 4/3, respectively, Find  $P(X \ge 1)$ .

[04 Marks]

**Q2.** Define correlation coefficient ( $\rho$ ) between two random variables X and Y. If X and Y are random variables and a, b, c, d are any numbers provide only  $a \neq 0, c \neq 0$ , then

$$\rho(aX + b, cY + d) = \frac{ac}{|ac|} \rho(x, Y)$$
[04 Marks]

Q3. If the probability that a person will believe a rumour is 0.6, find the probability that the tenth person to hear the rumour will be the third person to believe. [04 Marks]

Q4. State Little's law in Queuing theory. A monitor on a disk server showed that the average time to satisfy an input/output (I/O) request was 100 miliseconds. The I/O rate was about 100 requests per second. What was the mean number of requests at the disk server? [04 Marks]

Q5. A random variable X is said to have geometric distribution. Determine it's mean and variance.

#### [06 Marks]

**Q6.** An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4} , x > 1\\ 0 , elsewhere \end{cases}$$

(a) Verify that this is a valid density function.

(b) Evaluate cumulative distribution function F(x).

(c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

### [02+02+02=06 Marks]

**Q7.** Arrivals of plane at an airport follows Poisson process with a mean rate of 20 per hour. The airport can land 60 planes per hour, on an average, in good weather and 30 planes per hour in bad weather with a Poisson service rate. When there is congestion, the planes are forced to fly over the field in the stack awaiting landing.

- (a) How many planes should be flying over the field in the stack on an average in good weather and in bad weather?
- (b) How long a plane would be in the stack on the average in good weather and in bad weather?

#### [03+03=06 Marks]

**Q8.** Assume that the normal heart-rate in healthy individuals is normally distributed with an average 70 beats/minute with a standard deviation of 10 beats/minute. Out of 500 such healthy individuals, calculate the number of individuals whose hearts will beat at the rate (a) Over 88 beats/minute.

(b) Below 63 beats/minute.

(c) What is the heart-rate (to the nearest integer) beyond which 15% of the individuals fall?

(d) If, according to medical science, a "concerned zone" is defined as the heart-rate beyond  $\mu - 3\sigma$  and  $\mu + 3\sigma$ , then how many of these individuals fall into the "concerned zone"?

# [02+02+02+02=08 Marks]

Q9. In a certain market there are three brands of phones A, B and C. Given that a person last purchased phone of brand A, there is 70% chance that he would continue with Brand A, 20% and 10% chances that he would shift to brands B and C, respectively. Given that a person last purchased a phone of Brand B, there is 50% chances that he would shift to brand A and 10% chance to brand C. Given that a person last purchased a phone of Brand B, there is 50% chances d a phone of Brand A and 10% chance to brand C. Given that a person last purchased a phone of Brand C, there is 60%, 20% chance that he would shift to Brand A and B, respectively. The present market share of three brands A, B and C is 60%, 30% and 10% respectively. Using this information,

- (a) Find the probability that a customer who is currently a purchaser of brand A will purchase brand C after two time periods.
- (b) Find the probability that a customer will purchase brands A, B and C three period from now.
- (c) Find the market share of the brands, A, B and C in steady state.

## [02+02+04=08 Marks]

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