

2/12/2022
 (m)

Time: 3:00 hrs

Max. Mark: 50

Note: All questions are compulsory.

1. Define the following terms with examples: [1 × 5 = 5]
 - (i) Monotone function
 - (ii) Join of two Boolean matrices.
 - (iii) Eulerian graph
 - (iv) Complete bipartite graph
 - (v) Isomorphic graphs
2.
 - (i) A connected, planar graph contains 10 vertices and divides the plane into seven regions. Compute the number of edges in the graph. [2]
 - (ii) Explain with example how trees are different from graphs. [2]
 - (iii) Show that $K_{3,3}$ is non-planar. [2]
 - (iv) One of the applications of Fibonacci numbers is to convert kilometer to mile and vice-versa. Explaining the reason briefly, find kilometer equivalent of 50 miles. [2]
 - (v) Solve the recurrence relation given by [2]

$$a_n = a_{n-1} + \frac{n(n+1)}{2}, a_1 = 1.$$

3. Prove that an equivalence relation on a set partitions the set into equivalence classes and every partition of a set induces an equivalence relation on it. [5]
4. Solve the linear non-homogeneous recurrence relation with constant coefficients [5]

$$a_{n+3} - a_{n+2} - 4a_{n+1} + 4a_n = 3^n, n \geq 0,$$

where $a_0 = 0, a_1 = 1, a_2 = 2.$

5. Using the laws of logic, simplify the boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$, by quoting every law used. [5]
6. Find the number of positive integers below 3000 and not divisible by 7 or 8. [5]
7. Draw the Hasse diagram for the poset $(A, |)$, where $A = \{1, 2, 3, 6, 8, 12\}$ and $|$ denotes the divisibility relation. Find the minimal, maximal, least and greatest elements. [5]
8. Using generating functions, solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, where $a_0 = 2$ and $a_1 = 3$. [5]
9. A pet shop owner just received a shipment of 10 species of tropical fish. Since some species are incompatible, that is, they fight with some other species, they cannot be kept in the same tank. The following table summarizes the incompatibility of the various species, where a dot in row i and column j indicates species i and j are incompatible. Draw the graph of the problem. Determine the minimum number of tanks needed to store the fish and also find a possible way of storing them among those tanks. [5]

Species	1	2	3	4	5	6	7	8	9	10
1		
2			
3
4			.			.	.			
5	.	.					.			
6				
7
8	.		.			.				
9		.								
10	.		.				.			