Dr Sunil

MSMC

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28/02/2023

Roll No.:...............

National Institute of Technology Hamirpur (HP)

End Semester Examination

[Class: B.Tech. (1^{st} Semester)]

Title of the Course: Engineering Mathematics-I

Course Code: MA-111

Time: 3 Hours | *Maximum Marks: 50*

Note : Attempt all questions.

Q.No.1.: Obtain Fourier series for the function f(x) given by **(3+2)**

$$f(x) = 1 + \frac{2x}{\pi}, \quad -\pi \le x \le 0,$$

$$= 1 - \frac{2x}{\pi}, \quad 0 \le x \le \pi,$$

and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$.

Course Outcomes: CO1, CO3, CO4, CO6.

Q.No.2.: **(2+2+1)**

Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. Show that the equation is satisfied by A and hence obtains the inverse of the given matrix.

Course Outcomes: CO1, CO3, CO5, CO6.

Q.No.3.: Suppose u and v are functions of x and y defined by the relations **(5)**

$x = u + e^{-v} \sin u$, $y = v + e^{-v} \cos u$, then prove that $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$.

Course Outcomes: CO1, CO3, CO6.

Q.No.4.: **(5)**

Determine a, b, c such that $\operatorname*{Lt}_{\theta \to 0} \frac{\theta(a + b\cos\theta) - c\sin\theta}{\theta^5} = 1$.

Course Outcomes: CO1, CO3, CO6.

Q.No.5.: Suppose a closed rectangular box has length twice its breadth and has constant volume V. Determine the dimensions of the box requiring least surface area (sheet metal) by using Lagrange's method. **(5)**

Course Outcomes: CO1, CO2, CO3, CO6.

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Q.No.6.: Evaluate $\iint_R (x+y)^2 dxdy$, where R is the parallelogram in the xy-plane with vertices (1, 0), (3, 1), (2, 2), (0, 1) using the transformation $u = x + y$ and $v = x - 2y$. **(5)**

Course Outcomes: CO1, CO2, CO6.

Q.No.7.: By transforming into cylindrical coordinates evaluate the integral $\iiint (x^2 + y^2 + z^2) dxdydz$ taken over the region $0 \le z \le x^2 + y^2 \le 1$. **(5)**

Course Outcomes: CO1, CO2, CO3, CO6.

Q.No.8.: Find the magnitude of the directional derivative of the function $f(x, y) = x^2 + 3y^2$, in the direction normal to the circle $x^2 + y^2 = 2$, at the point $P = (1, 1)$. **(5)**

Course Outcomes: CO1, CO2, CO3, CO6.

Q.No.9.: Evaluate $\int_S \mathbf{F}.d\mathbf{S}$, where $\mathbf{F} = x\hat{\mathbf{I}} + (z^2 - zx)\hat{\mathbf{J}} - xy\hat{\mathbf{K}}$ and S is the triangular surface with vertices $(2, 0, 0), (0, 2, 0)$ and $(0, 0, 4)$. **(5)**

Course Outcomes: CO1, CO3, CO6.

Q.No.10.: Evaluate by Stoke's theorem $\int_C \mathbf{F}.d\mathbf{R}$ where $\mathbf{F} = y\hat{\mathbf{I}} + xz^3\hat{\mathbf{J}} - zy^3\hat{\mathbf{K}}$, C is the circle $x^2 + y^2 = 4$, $z = 1.5$. **(5)**

Course Outcomes: CO1, CO2, CO3, CO6.

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