Dr Bharat Bhushan Sharma 8/12/2021

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Roll No.:..............

National Institute of Technology, Hamirpur (HP)

Name of the Examination: B.Tech. (End Semester Examination)			
Branch	: Electrical Engineering	**Semester**	: 7th
Course Name	: Optimal Control Theory (Professional Elective-II)	**Course Code**	: EE – 452

Time: 3 Hours **Maximum Marks: 50**

Note: Attempt all the questions. Assume any missing data, appropriately.

Q.No.1: Explain fixed end problem and the associated necessary conditions for optimization. Find out extremum of following functional subjected to boundary conditions specified as $x_1(0) = x_2(0) = 1$ & $x_1\left(\frac{\pi}{4}\right) = x_2\left(\frac{\pi}{4}\right) = 2$, repectively:

$$J(x_1, x_2) = \int_0^{\frac{\pi}{4}} (x_1^2 + \dot{x}_2^2 + 2\dot{x}_1\dot{x}_2)\, dt$$

(7)

Q.No.2: What is static and dynamic optimization? Find points in 3-dimensional space to extremize the following function:

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 2x_1\, x_3$$

and which lie on intersection of following surfaces:

$$x_1 + 2\,x_2 + x_3 = 0; \quad 2x_1 - x_2 = 2 + x_3.$$

Also find the optimum value of the function.

(7)

Q.No.3: Show that Euler-Lagrange equation can be represented in following alternative form:

$$V_x - V_{t\dot{x}} - V_{x\dot{x}}\dot{x}^*(t) - V_{\dot{x}\dot{x}}\ddot{x}^*(t) = 0$$

Also, find the optimum of following function that satisfies the boundary conditions $x(0) = 1$ and $x(2) = 4$:

$$J(x) = \int_{t_0}^{t_f} (-t.x(t) + \dot{x}^2(t))\, dt$$

(7)

Q.No.4: Draw procedural summary of Hamiltonian approach of finding optimal control. Given a second order control system as follows:

$$\dot{x}_1(t) = -2x_2(t)$$
$$\dot{x}_2(t) = u(t)$$

The performance index is given as follows:

$$J = \frac{1}{2}\int_0^2 \{u^2(t)\}\, dt$$

Compute the optimal feedback control law using Hamiltonian approach when initial conditions are given as follows:

$$x_1(0) = 1; \; x_2(0) = 1 \;\&\; x_1(2) = 5; \; x_2(2) = 5.$$

(7)

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Q.No.5: For a first order system given as

$$\dot{x} = x + u$$

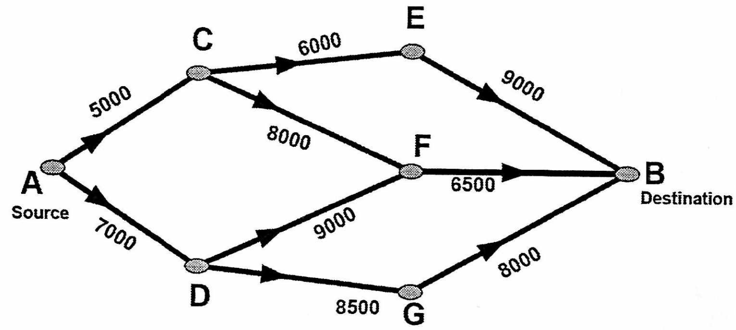
the performance index is given as

$$J(x) = \frac{1}{2}\int_0^{t_1} (x^2(t) + \frac{1}{2}u^2(t))\,dt$$

Find out the optimal feedback closed loop controller for the above case using Linear Quadratic Regulator (LQR) framework.

(7)

Q.No.6: Explain principle of optimality popularly used in Dynamic Programming? Also express the functional equation of the Dynamic Programming. A travelling salesman is required to move from his home station A to the destination station B to sell his product. Suggest the optimal control strategy using principle of optimality for the salesman if he has to plan his journey while keeping in view the possible routes to destination B through intermediate stations C, D, E, F & G. The unit costs involved in movement from one station to the other are specified along the links between the stations as per the following figure:



Also specify the optimal route and cost involved if salesman is already at station D.

(7)

Q.No.7: Explain any two of the following:

(a) Hamilton-Jacobi-Bellman (HJB) procedure for solving continuous time nonlinear optimal control problem using principle of optimality

(b) Time optimal control problem (TOCP) with input constraints and its implementation as a bang-bang controller

(c) Pontryagin Minimum Principle of optimization

(d) Procedural Summary of Linear Quadratic Regulator (LQR) control scheme

4+4=(8)
