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National Institute of Technology Hamirpur
Wavelet Transform & its Applications (EC-673)

Note: All questions are compulsory and carry equal marks. Write answers of sub-parts of each question in continuation only.

Maximum time: 3 Hours

- Q1. Consider the function $x(t) = t^2$ on $[0, 1]$ and $x(t) = 0$ elsewhere. Obtain the projection of x on space V_1 and compute four expansion coefficients.
- Q2. If $x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}$, then show that $X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$ Also show that the set of vectors $\{e^{j2\pi kn/N}\}$ form an orthogonal set.
- Q3. Define the regular distribution and singular distribution Dirac delta δ. Obtain Fourier transform of the function $x(t) = e^t$ in distributional sense.
- Q4. Write analysis and synthesis equations of the following representations using inner product notation: (a) Fourier series (b) Fourier transform (c) Discrete time Fourier transform (d) Discrete Fourier transform (e) Short-time Fourier transform (f) Continuous wavelet transform, and (g) Discrete wavelet transform.
- Q5. Define the Harr scaling, $\phi(t)$, and wavelet, $\psi(t)$, functions and obtain their Fourier transform. Plot the function $x(t) = 2\phi(4t) + 2\phi(4t - 1) + \phi(4t - 2) - \phi(4t - 3)$, and write this function in terms of the basis functions of the V_0, W_0, and W_1 spaces.
- Q6. Define space $L_2(\mathbb{R})$ of functions and show that $L_2(\mathbb{R}) = V_0 \oplus W_0 \oplus W_1 \oplus W_2 \oplus \ldots = V_0 \oplus \left[\bigoplus_{j=0}^{\infty} W_j\right]$.
- Q7. Explain multi-resolution analysis (MRA) using axioms of MRA. Let $\{V_j\}$ be an MRA with scaling function ϕ, then show that the set of functions $\{\phi_{jk}(t) = 2^{j/2}\phi(2^j t - k),\ j, k \in \mathbb{Z}\}$ forms an orthogonal basis of the subspace V_j.
