

Time: 3 Hrs.

Max. Marks: 50

Note: The symbols and variables used have their usual meaning. All questions carry equal marks.

- Q1. (a) If $x(t) = e^{j(2t+\pi/4)}$, then determine the normalized energy and normalized average power of $x(t)$. Hence, classify it as a power or an energy signal.
(b) Find the Fourier transform of the following signals
(i) $x_1(t) = \cos(\omega_0 t)$ (ii) $x_2(t) = \frac{1}{1+jt}$
- Q2. (a) A continuous random variable has a probability density function given by $f_X(x) = kx^2$, for $0 \leq x \leq 1$ and zero elsewhere. Find values of the constants k and a such that $P(X \leq a) = P(X > a)$.
(b) Find the expected values $E(X)$ and $E(X^2)$ of a random variable X whose probability density function is given by $f_X(x) = (1-x)^2$, for $0 \leq x \leq 1$ and zero elsewhere.
- Q3. (a) Consider a wide-sense stationary (WSS) random process $X(t)$ being applied to the input of an LTI system whose impulse response is $h(t) = 3e^{-2t}u(t)$. Find the mean value of the output random process $Y(t)$ of the system if $E[X(t)] = 2$.
(b) A WSS random process $X(t)$ with autocorrelation $R_{XX}(\tau) = e^{-2|\tau|}$ is applied to the input of an LTI system with impulse response $h(t) = e^{-2t}u(t)$. Find the power spectral density of the output random process $Y(t)$ of the system.
- Q4. (a) Discuss the following types of noise in communication systems
(i) Shot noise (ii) Thermal noise
(b) An amplifier has a bandwidth of 500 kHz, and an input resistance of 50 Ω . When a 0.5 μ V input signal level is applied to the amplifier input under matched conditions, the output signal-to-noise ratio (SNR) = 0 dB. Determine the noise figure of the amplifier. Assume room temperature of 290 K.
- Q5. (a) Discuss the following discrete memoryless channels in terms of their channel matrix
(i) Lossless channel (ii) Deterministic channel
(iii) Noiseless channel (iv) Binary symmetric channel
(b) Show that for a binary symmetric channel (BSC), the mutual information is given by
$$I(X; Y) = H(Y) + p \log_2 p + (1-p) \log_2 (1-p),$$
where X is the channel input alphabet, Y is the channel output alphabet, and p is the channel transition probability.