

National Institute of Technology, Hamirpur
M.Sc. End Semester Examination, December 2020

Subject Name : Real Analysis Semester : 1st
Subject Code : MA - 611 Full Marks : 50
Department : Mathematics & Scientific Computing Time : 2 Hour

Note : All questions are compulsory .

(1) Prove that every compact subset of a metric space is closed. (8)

(2) State and prove Cauchy's Second Theorem on limit. (6)

(3) State and prove Abel's Test for the convergence of improper integral. (6)

(4) State and prove Dini's Test for the uniform convergence of a sequence of real valued function. (6)

(5) Test the convergence of $\int_0^{\infty} \frac{x}{1+x^4 \cos^2 x} dx$. (8)

(6) If a function $f(x)$ is continuous on $[0, 1]$ then estimate

$$\lim_{x \rightarrow \infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} dx. \quad (8)$$

(7) Test the uniform convergence of the series $\sum \{\log(n+1)\}^{-x} \cos nx$ in $[\theta_1, \theta_2]$. $0 < \theta_1 \leq x \leq \theta_2 < 2\pi$. (8)