National Institute of Technology Hamirpur
Department of Chemical Engineering
M. Tech.: 1st Semester
End Semester Theory Examination
CH-611: Advance Transport Phenomena

Date: 14th December 2020 Time: 11:00 to 13:00 Max. Marks: 50

Note: *(i) Answer all the questions (ii) Write proper valid assumptions, (iii) while solving the problems intermediate steps are very-very important, (iii) Make necessary assumptions for missing data, if any.*

Q.1 Show that: 5

(i) $\nabla \times (\nabla \times \mathbf{u}) = \nabla(\nabla \cdot \mathbf{u}) - \nabla^2 \mathbf{u}$

(ii) $\nabla \cdot (\nabla \times \mathbf{u}) = 0$

(iii) $S_{ij}A_{ij} = 0$ where S_{ij} and A_{ij} are a symmetric and antisymmetric matrix respectively.

Q.2 Write the divergence of the dyad $\rho\boldsymbol{vv}$ in index notation. Expand the derivatives using the chain rule. Write the continuity equation in index notation and use this in the expanded expression for the divergence of the above dyad. Simplify and show that the result is $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$. 5

Q.3 The Laplacian of a scalar is important in mass and heat transfer applications, and is obtained by applying the operator $\nabla \cdot$ the gradient of a scalar, e.g., to ∇T. By applying this to $\nabla = \frac{\partial}{\partial x} e_x + \frac{\partial}{\partial x} e_y + \frac{\partial}{\partial x} e_z$ verify that the Laplacian in Cartesian coordinates is given by 5

$$\nabla \cdot \nabla T = \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial y^2}$$

Q.4 The stresses acting on the faces of a cube of unit length in all three directions are as follows: 5

$$T_{ij} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

where *i* denotes the direction of the force and *j* denotes the direction of normal to the area. Find out the force acting on the faces of the cube in the three directions.

Q.5 Consider the parabolic flow of a fluid in a tube of radius R, with the velocity given in cylindrical coordinates by: 10

$$v_z = V\left(1 - \frac{r^2}{R^2}\right)$$

Separate the rate of deformation tensor into its elementary components.

Q.6 Consider the uniform flow of a fluid past a flat plate of infinite extent in the $x_1 - x_3$ plane, with the edge of the plate at $x_1 = 0$. The Reynolds number based on the fluid velocity and the length of the plate is large. At a large distance from the plate, the fluid has a uniform velocity U_1 in the x_1 direction and U_3 in the x_3 direction. All velocities are independent of the x_3 direction, and 10 the velocities U_1 and U_3 are uniform and independent of position in the outer flow. Write down the equations of motion in the x_1, x_2 and x_3 directions. If the thickness of the boundary layer δ is small compared to the length of the plate L, find the leading order terms in the conservation equations.

Q.7 Consider an irreversible, bimolecular reaction in a liquid film at steady state with species conservation as shown below: 10

$$\frac{\partial^2\theta}{\partial\eta^2} = Da\theta(\theta + 2\eta - 1)$$

$$\theta(0) = 1, \theta(1) = 0$$

where $\theta = C_A/C_{A0}$ $\eta = x/L$ and $Da = KC_{A0}L^2/D_A$. For slow reaction kinetics $(Da \ll 1)$a regular perturbation scheme can be used. Determine $\theta(\eta)$for slow kinetics, including terms of $O(Da)$.